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Diffusion in a heterogeneous medium – or how voting preferences can be described in terms of physics

A drop of dye added to a glass of water undergoes ordinary diffusion. However, when placed on the surface of a foam, the dye spreads differently – diffusion becomes anomalous. An example of this is the pattern on the froth of a cup of cappuccino. Interestingly, the latest research suggests that diffusion equations in a heterogeneous environment can also describe social phenomena, such as election results or the behaviour of stock market traders.

The movement of particles in complex media – such as porous materials, gels or foams – bears more resemblance to a random journey through an irregular maze than to a leisurely stroll through a homogeneous space. The presence of local 'traps' alongside narrow passages or branches causes the transport of matter or energy to be significantly slowed down or accelerated. Such deviations from classical diffusion are referred to as anomalous diffusion. It is also observed in media with a non-uniform structure. An international team of physicists from Poland, Croatia, Macedonia and Hungary has undertaken a mathematical description of diffusion in such systems; the Polish side was represented by scientists from the Institute of Nuclear Physics of the Polish Academy of Sciences (IFJ PAN) in Cracow.

We usually speak of diffusion when certain physical entities (such as atoms, chemical molecules, dye particles, or even thermal energy) move from an area of higher concentration to an area of lower concentration as a result of random interactions with their surroundings. A classic example of simple diffusion is the familiar process of a drop of dye spreading out in a glass of still water.

"In the simplest models, it is assumed that the diffusion coefficient – which determines how a particle moves – is the same at every point in space. My team addressed the problem of diffusion in a heterogeneous medium, where the diffusion coefficient varies spatially. An example of such a situation is a glass containing a mixture of liquids with density varying spatially. The problem of describing diffusion in such a medium boils down to solving a modified diffusion equation," explains Prof. Katarzyna Gorska (IFJ PAN), the lead author of an article published in the interdisciplinary journal *Chaos*.

A similar phenomenon can be observed in nature in many contexts, including the way bacteria move, the transport of molecules across cell membranes, in heat propagation in heterogeneous materials, in the movement of charge carriers in semiconductors, or even in the transmission of information within a crowd, voter behaviour or the reactions of financial markets.

"The classical diffusion equation is widely used because of the mathematical ease with which its solutions can be applied. Despite its good agreement with reality, this equation has a non-physical feature: the diffusing particles propagate instantaneously. In our research, we modified the basic equations to obtain a finite particle propagation velocity. This leads to a hyperbolic equation, known as the telegraph equation, which describes phenomena occurring in transmission lines," notes Prof. Andrzej Horzela (IFJ PAN).

The solutions obtained by the researchers for particles diffusing at a finite velocity turned out to be solutions to the Cattaneo-Vernotte equation, which resembles the telegraph equation but satisfies physical conditions suited to describing diffusion. These were then analysed for cases where the diffusion coefficient varies with position (for the sake of simplicity, the model was one-dimensional), and solutions were proposed for specific diffusion coefficient models.

The team of researchers noted that the resulting equations, describing physical anomalous diffusion in heterogeneous media, bear a striking mathematical resemblance to the equation used to model shifts in public opinion. The analogy relates to the so-called 'voter with noise' model, where it is assumed that voters generally adopt the opinions of their neighbours (i.e. follow the herd), but there are also voters capable of spontaneously changing their minds (this effect acts as noise). The observed similarity suggests that the mechanisms of anomalous diffusion in heterogeneous physical systems and the mechanisms of opinion propagation in social structures, at least under certain conditions, appear to be of a similar nature.

The analyses also suggest that the behaviour of financial markets moving towards or returning to equilibrium in situations where investors conceal their intentions may also exhibit the characteristics of anomalous diffusion in a heterogeneous environment.

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The Henryk Niewodniczański Institute of Nuclear Physics (IFJ PAN) is currently one of the largest research institutes of the Polish Academy of Sciences. A wide range of research carried out at IFJ PAN covers basic and applied studies, from particle physics and astrophysics, through hadron physics, high-, medium-, and low-energy nuclear physics, condensed matter physics (including materials engineering), to various applications of nuclear physics in interdisciplinary research, covering medical physics, dosimetry, radiation and environmental biology, environmental protection, and other related disciplines. The average yearly publication output of IFJ PAN includes over 600 scientific papers in high-impact international journals. Each year the Institute hosts about 20 international and national scientific conferences. One of the most important establishments of the Institute is the Bronowice Cyclotron Centre (CCB), which is an infrastructure unique in Central Europe, serving as a clinical and research centre in the field of medical and nuclear physics. In addition, IFJ PAN runs four accredited research and measurement laboratories. IFJ PAN is a member of the Marian Smoluchowski Kraków Research Consortium: "Matter-Energy-Future", which in 2012-2017 enjoyed the status of the Leading National Research Centre (KNOW) in physics. In 2017, the European Commission granted the Institute the HR Excellence in Research award. As a result of the categorization of the Ministry of Education and Science, the Institute has been classified into the A+ category (the highest scientific category in Poland) in the field of physical sciences.

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SCIENTIFIC PUBLICATIONS:

"Heterogeneous Cattaneo-Vernotte equation connection to the noisy voter model"
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Chaos 36, 043108 (2026)
DOI: 10.1063/5.0325574

LINKS:

http://www.ifj.edu.pl/
The website of the Institute of Nuclear Physics, Polish Academy of Sciences.

http://press.ifj.edu.pl/
Press releases of the Institute of Nuclear Physics, Polish Academy of Sciences.

IMAGES:

IFJ260610b_fot01s.jpg **HR:** http://press.ifj.edu.pl/news/2026/06/10/IFJ260610b_fot01.jpg
The milky pattern on the surface of the foamed coffee provides a delicious example of… anomalous diffusion in an inhomogeneous medium. (Source: IFJ PAN)